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A flow-network approach for equilibrium of material requirements planning

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Abstract

Material requirements planning (MRP) has been a very popular and widely used multi-level inventory control method since 1970s. Recent developments in computer and information technology accelerate and facilitate the calculations necessary for MRP, but MRP is simply a system to open and trace the production/purchasing orders under pre-determined lead-time and lot size constraints. It does not directly include any optimization feature. In this article, an approach, which consists of the Flow Network with Side Constraints, is discussed in order to optimize the material flows in MRP problems. Additionally, an example case is given in order to show the applicability of the flow network formulation to APS. The model of the example case is solved and the computational results are given.

Keywords: Material requirements planning (MRP); Flow network; Optimization; Side constraints

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1. Introduction

Inventory management is one of the most important functions of a production system. A production system or a manufacturing company has two types of inventory as defined by Plossl (1994) and Orlicky (1975). While the first type—manufacturing inventory—consists of raw materials, semi-finished component parts, finished component parts, sub-assemblies, component parts in process and sub-assemblies in process, the second one—the distribution inventory—is made up of completed products in warehouses and completed products in transit. The main goal of a company is to obtain profit by meeting costumers' demands completely and on time. To meet demands completely and on time requires a good inventory management supporting the production planning and keeping inventories under control. According to Orlicky (1975), inventory management is based on one of the two different approaches: (1) stock

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replenishment or statistical inventory control which is based on monitoring inventory levels according to the policy preferred in order to eliminate the probability of being shortage by statistically analyzing the consumption rates (2) material requirements planning (MRP) which is a method based on planning the requirements according to the master production schedule (MPS) which is prepared depending on customers' demands. Two basic data are necessary for MRP: (1) the MPS, and (2) Bill of materials (BOM).

MPS is a plan showing the product which will be produced when and in what quantity based on forecasting or received customer orders. BOM shows which sub-component or raw material is used for which product and in what quantity. Required material quantities are calculated by hierarchically multiplying the production quantities in MPS by unit usage coefficients in BOM.

2. Current state of literature for MRP

Rondeau and Litteral (2001) give a brief summary about the historical improvements of manufacturing planning and control systems. Segerstedt (1996) presents formulas, which are used for calculations of MRP. Formulation is discussed in detail and a flow chart for the method is given. However, the formulation given has no optimization property. Chu (1995) intensifies on aggregate optimality. The objective for the model is to maximize the total (potential) profit. An LP model has been developed subject to the restrictions on the supply, the total demand and the labor resource. Plenert (1999) compares MRP with just in time (JIT), optimized production technology (OPT)/theory of constraints (TOC) and bottleneck allocation methodology (BAM), and claims its strength in job shops. The incorrect uses of MRP system are also discussed. According to the author, the basic abuses of the MRP environment is the lead time which is a kind of non productive time made up of elements like queue time, waiting time, transfer time, etc. Euwe and Wortmann (1997) discuss the deficiencies of MRP in issues like flexibility in lot sizes, product structures' capacity constraints and alternative plans. They also define 'vision statements' for the future planning systems in order to overcome such difficulties. Van Donselaar and Gubbels (2002) compare MRP and line requirements planning (LRP) for planning orders. Their research basically focuses on minimizing the system inventory and system nervousness. They also discuss and propose LRP technique to achieve their goals. Zijm (2000) discusses a framework emphasizing the integration of technological and logistics planning, capacity planning and materials coordination issues. He also classifies advances in manufacturing practice in three different fields: hardware automation, design and process planning and manufacturing planning and control systems, and then, adds system complexity reduction as a fourth impact factor. Zijm (2000) also emphasizes that OR models are being integrated in planning systems of some software manufacturers. Clark (2003) proposes three mixed integer programming models in his study.

The network flow models can be solved more efficiently and faster than those developed in general linear form. McBride (1985) develops an algorithm, discusses it in details and gives computational comparisons of the code of his program called "EMNET" with "MINOS" of IBM. He finds that network approach and solution is about five times faster than MINOS. Ali et al. (1988) define an algorithm for network problems, which have side constraints to assure some arcs having equal flow and give computational results of their code "EQFLO" by comparing "MSPX" of IBM. Mathies and Mevert (1998) give some additional computational results from the literature while discussing their algorithms. Glover et al. (1992) give some applications of the pure flow networks to the inventory problems as the examples for the dynamic network models. They apply the pure flow network approach to the multi-period inventory problems. They also extend their models to include backorders, multi-products and multi-plants. However, they did not take the multi-level hierarchical product tree and lead times into consideration.

Yenisey (1998) gives a primitive study for the network optimization of MRP. However, that study has some deficiencies to express all the aspects of an MRP system. Hence, an improved optimization model, based on pure flow network with side constraints formulation, is developed in this research. Since, the given model is based on the flow network approach, the research has an optimization feature. The model is constructed in a manner of expressing the product tree structure (i.e. BOM) by the side constraints and including the lead times of the components and the raw materials by the arrangements made on the indices. Similarly, the lead times of the products can be added in the model. MRP approach has a drawback because of fixed lot sizes. The model discussed in this research deals with such a drawback by applying the lot-for-lot policy. The previously ordered lots can easily be added into model by defining them as inventories from previous periods and rewriting the model accordingly in order to overcome the nervousness problem. Additionally, the recalculations after such modifications will always lead to minimization of on-hand inventory levels unless the user adds lower bound constraints on flows for inventories carried to following periods to avoid shortages or to express buffer inventories. Furthermore, another objective for this research is to gain the advantages of the flow network algorithms because of solution times.

3. Model

First of all, it is necessary to explain the period indexing of the time-dependent notation. The initial period of the planning horizon and the total number of periods in the planning horizon may differ from one problem to another. Therefore, it is preferred to define the period indices in the "set of periods in planning horizon", I. Additionally, the full formulation of the example case is given in the appendix in order to clear the model.

The model that is developed for this paper is given and discussed below. Firstly, it is necessary to discuss the flow network given in Fig. 1 in order to make the model clearer. As it is seen in the figure, it is a typical flow network with side constraints. All physical materials flows are expressed as network flows. Nodes represent products, sub-components and raw materials. It is preferred to develop the network in a pure network form. However, it can be easily developed in a generalized network form by simply modifying the network as adding arc multipliers in order to express gross usage or to include the materials to be ruined or damaged in the warehouses. It is chosen to use side constraints to express the transformation of usage of an item to a sub-item. In the figure, only one period is shown. It is easy to show whole planning horizon by repeating the network as the number of periods in the planning horizon. This network is made of the relations in bill-of-materials. The network can be interpreted at three hierarchical levels. The first level is for products, second is for sub-components, and finally, the third one is for raw materials.

A three-level hierarchical product tree made of products, sub-components and raw materials is used for the research. Product supplies in every period of planning horizon trigger the system as pulling required products in order to satisfy the demands. Sub-components according to these supplies and raw materials depending on sub-components flow through the network. The plant may have some initial inventory on hand and a policy of transferring some inventory from a period to others following. An upper bound made of zero can be put on these flows in order not to allow on-hand inventory in stock policy. The initial on-hand inventories' flows have upper bounds representing the amounts being held in stock. If the decision-maker wants to hold inventory at the end of planning horizon, he/she can put lower bounds on on-hand inventory flows going beyond the last period. Although there are several lot-sizing policies defined in the literature, this model is based on lot-for-lot policy as Orickly (1975) defines.

The parameter k_r must be defined as follows for each raw material before the formulation is given.

$$k_r = \max\left\{\mathrm{CL}_c \middle| (c, r) \in \mathrm{BOMC}\right\}.$$

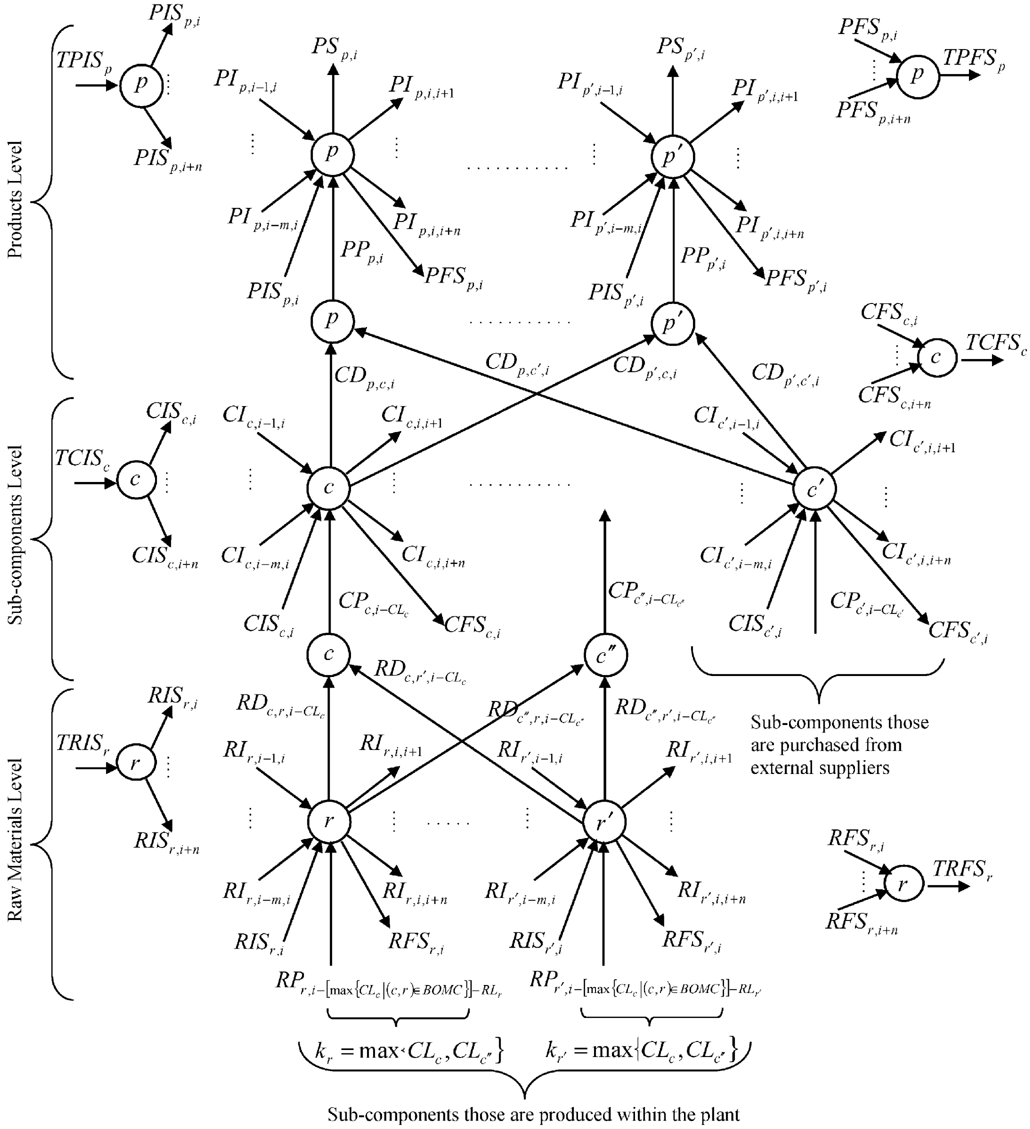

Fig. 1. The flow network of materials requirement planning system.

Mathematical model of the flow network given in Fig. 1 is as follows.

$$\min \sum_{p\in P}\sum_{i\in I}\mathrm{pc}_p\mathrm{PP}_{p,i}+\sum_{c\in C}\sum_{i\in I}\mathrm{cc}_c\mathrm{CP}_{c,i-\mathrm{CL}_c}+\sum_{r\in R}\sum_{i\in I}\mathrm{rc}_r\mathrm{RP}_{r,i-k_r-\mathrm{RL}_r}$$
$$+\sum_{p\in P}\sum_{i\in I-\{\max\ I\}}\sum_{\substack{j\in I\\(j>i)}}[(j-i)\mathrm{ph}_p]\mathrm{PI}_{p,i,j}+\sum_{c\in C}\sum_{i\in I-\{\max\ I\}}\sum_{\substack{j\in I\\(j>i)}}[(j-i)\mathrm{ch}_c]\mathrm{CI}_{c,i,j}$$

$$
\begin{aligned}
&+\sum_{r\in R}\sum_{i\in I-\{\max\, I\}}\sum_{\substack{j\in I\\(j>i)}}[(j-i)\mathrm{rh}_r]\mathrm{RI}_{r,i-k_r,j-k_r}+\sum_{p\in P}\sum_{i\in I}(\max\, I+1-i)\mathrm{ph}_p\mathrm{PFS}_{p,i}\\
&+\sum_{c\in C}\sum_{i\in I}(\max\, I+1-i)\mathrm{ch}_c\mathrm{CFS}_{c,i}+\sum_{r\in R}\sum_{i\in I}(\max\, I+1-i)\mathrm{rh}_r\mathrm{RFS}_{r,i-k_r}\\
&+\sum_{p\in P}\sum_{i\in I}(i-\min\, I)\mathrm{ph}_p\mathrm{PIS}_{p,i}+\sum_{c\in C}\sum_{i\in I}(i-\min\, I)\mathrm{ch}_c\mathrm{CIS}_{c,i}\\
&+\sum_{r\in R}\sum_{i\in I}(i-\min\, I)\mathrm{rh}_r\mathrm{RIS}_{r,i-k_r}+\sum_{p\in P}\mathrm{pih}_p\mathrm{TPIS}_p+\sum_{c\in C}\mathrm{cih}_c\mathrm{TCIS}_c+\sum_{r\in R}\mathrm{rih}_r\mathrm{TRIS}_r
\end{aligned}
\tag{1}
$$

Subject to

$$\mathrm{PS}_{p,i}+\sum_{\substack{j\in I\\(j>i)}}\mathrm{PI}_{p,i,j}+\mathrm{PFS}_{p,i}-\sum_{\substack{j\in I\\(j<i)}}\mathrm{PI}_{p,j,i}-\mathrm{PP}_{p,i}-\mathrm{PIS}_{p,i}=0,\quad p\in P,\ i\in I \tag{2}$$

$$\mathrm{PS}_{p,i}\geqslant\mathrm{PD}_{p,i},\quad p\in P,\ i\in I \tag{3}$$

$$\sum_{i\in I}\mathrm{PIS}_{p,i}-\mathrm{TPIS}_p=0,\quad p\in P \tag{4}$$

$$-\sum_{i\in I}\mathrm{PFS}_{p,i}+\mathrm{TPFS}_p=0,\quad p\in P \tag{5}$$

$$\mathrm{CD}_{p,c,i}-\mathrm{cu}_{p,c}\mathrm{PP}_{p,i}=0,\quad i\in I,\ (p,c)\in\mathrm{BOMP} \tag{6}$$

$$\sum_{(p,c)\in\mathrm{BOMP}}\mathrm{CD}_{p,c,i}+\sum_{\substack{j\in I\\(j>i)}}\mathrm{CI}_{c,i,j}+\mathrm{CFS}_{c,i}-\sum_{\substack{j\in I\\(j<i)}}\mathrm{CI}_{c,j,i}-\mathrm{CP}_{c,i-\mathrm{CL}_c}-\mathrm{CIS}_{c,i}=0,\quad c\in C,\ i\in I \tag{7}$$

$$\sum_{i\in I}\mathrm{CIS}_{c,i}-\mathrm{TCIS}_c=0,\quad c\in C \tag{8}$$

$$-\sum_{i\in I}\mathrm{CFS}_{c,i}+\mathrm{TCFS}_c=0,\quad c\in C \tag{9}$$

$$\mathrm{RD}_{c,r,i-\mathrm{CL}_c}-\mathrm{ru}_{c,r}\mathrm{CP}_{c,i-\mathrm{CL}_c}=0,\quad i\in I,\ (c,r)\in\mathrm{BOMC} \tag{10}$$

$$
\begin{aligned}
&\sum_{(c,r)\in\mathrm{BOMC}}\mathrm{RD}_{c,r,i-\mathrm{CL}_c}+\sum_{\substack{j\in I\\(j>i)}}\mathrm{RI}_{r,i-k_r,j-k_r}+\mathrm{RFS}_{r,i-k_r}-\sum_{\substack{j\in I\\(j<i)}}\mathrm{RI}_{r,j-k_r,i-k_r}-\mathrm{RP}_{r,i-k_r-\mathrm{RL}_r}-\mathrm{RIS}_{r,i-k_r}=0,\\
&i\in I,\ r\in R
\end{aligned}
\tag{11}
$$

$$\sum_{i\in I}\mathrm{RIS}_{r,i-k_r}-\mathrm{TRIS}_r=0,\quad r\in R \tag{12}$$

$$-\sum_{i\in I}\mathrm{RFS}_{r,i-k_r}+\mathrm{TRFS}_r=0,\quad r\in R, \tag{13}$$

where:

Indexes:

p	index for products
i,j	index for periods

c	index for sub-components
r	index for raw materials

Costs:

pc_p	unit cost of product p including all production costs other than raw materials' and sub-components' purchasing costs
cc_c	unit cost of sub-component c
rc_r	unit cost of raw material r
ph_p	holding cost of product p
ch_c	holding cost of sub-component c
rh_r	holding cost of raw material r
pih_p	holding cost of product p for initial period
cih_c	holding cost of sub-component c for initial period
rih_r	holding cost of raw material r for initial period

Parameters:

$\mathrm{PD}_{p,i}$	demand for product p in period i
TPIS_p	total initial on-hand inventory level of product p
TCIS_c	total initial on-hand inventory level of sub-component c
TRIS_r	total initial on-hand inventory level of raw material r
TPFS_p	total final on-hand inventory level of product p
TCFS_c	total final on-hand inventory level of sub-component c
TRFS_r	total final on-hand inventory level of raw material r
$\mathrm{cu}_{p,c}$	unit usage of sub-component c for product p
$\mathrm{ru}_{c,r}$	unit usage of raw material r for sub-component c
CL_c	lead time of sub-component c
RL_r	lead time of raw material r.

Variables:

Basic decision variables:

$\mathrm{PP}_{p,j}$	quantity of product p that must be procured in period j
$\mathrm{CP}_{c,j}$	quantity of sub-component c that must be procured in period j
$\mathrm{RP}_{r,j}$	quantity of raw material r that must be procured in period j

Secondary decision variables:

$\mathrm{PI}_{p,i,j}$	inventory of product p carried from period i to period j
$\mathrm{CI}_{c,i,j}$	inventory of sub-component c carried from period i to period j
$\mathrm{RI}_{r,i,j}$	inventory of raw material r carried from period i to period j
$\mathrm{PIS}_{p,i}$	quantity of product p to be used in period i from initial on-hand inventory
$\mathrm{CIS}_{c,i}$	quantity of component c to be used in period i from initial on-hand inventory
$\mathrm{RIS}_{r,i}$	quantity of raw material r to be used in period i from initial on-hand inventory
$\mathrm{PFS}_{p,i}$	quantity of product p to be saved in period i for on-hand inventory beyond planning horizon

$\text{CFS}_{c,i}$	quantity of component c to be saved in period i for on-hand inventory beyond planning horizon
$\text{RFS}_{r,i}$	quantity of raw material r to be saved in period i for on-hand inventory beyond planning horizon

Side decision variables:

$\text{PS}_{p,i}$	supply of product p in period i
$\text{CD}_{p,c,i}$	demand for sub-component c to be used in product p in period i
$\text{RD}_{c,r,i}$	demand for raw material r to be used in sub-component c in period i

Sets:

P	set of products
C	set of sub-components
R	set of raw materials
I	set of periods in planning horizon
$\text{BOMP} = \{(p,c) \mid p \in P,\ c \in C,\ c \text{ is a sub-component used in product } p\}$	Set of Bill-of-materials for relations between sub-components and products
$\text{BOMC} = \{(c,r) \mid c \in C,\ r \in R,\ r \text{ is a raw material used in sub-component } c\}$	Set of Bill-of-materials for relations between raw materials and sub-components.

Objective function (1) includes not only production and purchasing costs but also holding costs. The cost of each item for all periods in planning horizon is included in objective function. All costs are incurred at the beginning of periods. Unit product cost pc_p includes all production costs other than sub-components' production costs and all purchasing costs. Unit sub-component cost cc_c is purchasing cost for items procured from outside while production cost for those produced within the plant. Holding costs for periods in the planning horizon, ph_p, ch_c and rh_r, include only inventory carrying cost. They do not include any production or purchasing cost. These holding costs are also valid for the inventories being saved for the periods beyond the last period of the planning horizon. However, the holding costs defined for the inventories being carried from the periods before the first period of the planning horizon, pih_p, cih_c and rih_r include all costs made of both production or purchasing cost and inventory carrying costs after they were procured. One can discuss why ph_p, ch_c and rh_r do not include purchasing or production costs. Because, production or purchasing costs are included while they are flowing into the network for all items entering the system during planning horizon. Therefore, if holding costs would have production or purchasing costs, they were being repeated. Moreover, it is clear that the first periods will not have incoming on-hand inventories other than initial stocks. Similarly, the last periods will not have outgoing on-hand inventories other than final stocks. Another important point is that the time value of money is not considered in the model. However, if the planning horizon spans over a long period of time, it would become important to take the time value of money into account. The interest rate can be added in the formulation as a multiplier to handle the time value of money.

It is necessary to underline that an adjustment is made for indices used in model in order to make them consistent, i.e. $\min I = \max\{\text{CL}_c + \text{RL}_r \mid (c,r) \in \text{BOMC}\} + 1$. This equation is used as a basis for the adjustments made in Eqs. (11) and (12). Hence, it is guaranteed that only positive indices are used and the first period of planning horizon's index is one. Otherwise, it may be obligated to use negative indices for expressing period index of raw materials. Moreover, indices are also used as multipliers in the formulation as Eq. (1). The possibility of an index having zero value will cause that to multiply by zero. Therefore, such an adjustment is necessary to avoid using the zero -multiplications.

In the first constraint (2), equilibrium among product supplies, on-hand inventories and products necessary to satisfy the demands of both customers and inventory policy are written in flow network form. The second constraint (3) is written in order to ensure that the demand of a product by customers will be satisfied. The constraint in (4) is put to guarantee that the sum of on-hand inventories coming from the previous periods of the planning horizon must equal to the total initial on-hand stocks. Similarly, Eq. (5) is written in order to assure that the sum of the inventories to be saved in each planning period for the beyond of the planning horizon must equal to the total on-hand stock that is predetermined as a policy. The Eq. in (6) is a side constraint to transform the materials flow from products to sub-components by multiplying products' flows by unit sub-component usages. The constraint in (7) is another flow conservation equation to find out sub-components' requirements. The Eq. (8) is to prove the limitation coming from initial on-hand inventories at the beginning of planning horizon, and the Eq. (9) is written for the closing on-hand inventories of the sub-components as explained in the Eq. (5) for products above. The constraints in (10)–(13) are for raw materials' usages, requirements and initial and final on-hand inventories, respectively, as the same manner explained in (6)–(9).

Although the fundamental aim of this study is to introduce a flow-network formulation for the solution of MRP problems, it would be better to give some idea about the size of the model and the effect of the length of planning horizon. Before the definition of the model's size, it is necessary to define the variables used for its calculation; i.e., nI, nP, nC and nR denote the number of periods, products, components and raw materials, respectively, $C(nI,2)$ denotes the number of combinations of nI object taken two at a time, $s(\mathrm{BOMP})$ and $s(\mathrm{BOMC})$ denote the number of element in sets BOMP and BOMC, respectively. The number of basic decision variables is $nI \times (nP + nC + nR)$. The number of secondary decision variables consists the holding inventories, the initial inventories and the final inventories. The number of the holding inventories is $C(nI,2) \times (nP + nC + nR)$, the number of the initial inventories is $nI \times (nP + nC + nR)$, and the number of the final inventories is again $nI \times (nP + nC + nR)$. The number of side decision variables is the sum of product, component and raw material usages. The number of side decision variables for products is $nP \times nI$, for components is $s(\mathrm{BOMP}) \times nI$ and for raw material is $s(\mathrm{BOMC}) \times nI$.

The number of the constraints and its distribution according to the type of the inventory item are as follows. The number of the conservation of flow constraints for products is $2 \times nP \times (nI + 1)$, that for components is $nC \times (nI + 2)$ and for raw materials it is $nR \times (nI + 3)$. Finally, the number of side constraints according to the unit usage for the product–component relations is $nP \times nI \times s(\mathrm{BOMP})$ and for the component–raw material relations is $nR \times nI \times s(\mathrm{BOMC})$. As seen from the calculations about the size of the model, it is obvious that the number of variables and constraints increase by the number of the periods in the planning horizon.

4. Example

A three-level hierarchical model made of two periods is given as an example for this research. The BOM structure for the example case is given in Fig. 2. The example consists of two products, three sub-components and two raw materials.

In Fig. 2, numbers in brackets by nodes represent the lead times for the items, and the numbers in parentheses near arcs represent the unit usage of the sub-items. The whole flow network for the example is given in Fig. 3. Additionally, the full formulation of the example is given in Appendix A.

The network structure given Fig. 3 almost reflects the BOM structure of the case given Fig. 2. This problem is made of three periods. Arcs representing the materials' flows among them connect the periods to each other. The network structure of the example has very high flexibility in order to include more periods, products and sub-items.

The model of the example case was solved by Hyper Lindo/PC running on a notebook consists of Athlon 2000+ CPU, 256 Mb RAM and 32 Mb Radeon video card. The optimum achieved at 73rd iteration with the objective function value 12209890. The model made of 81 rows and 130 variables. The number of nonzeroes was 364. The solution time was 0.42 s.

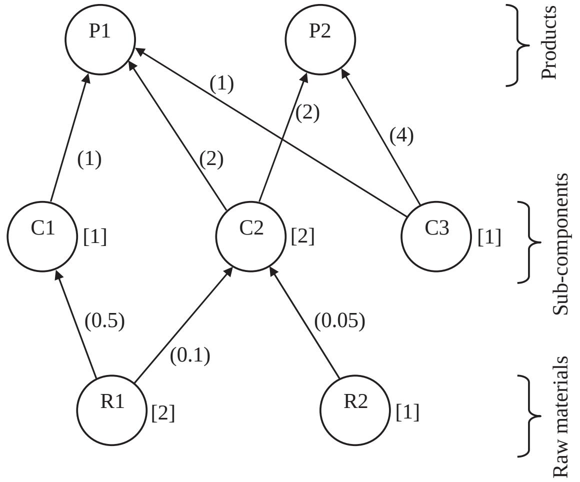

Fig. 2. The BOM of the example case.

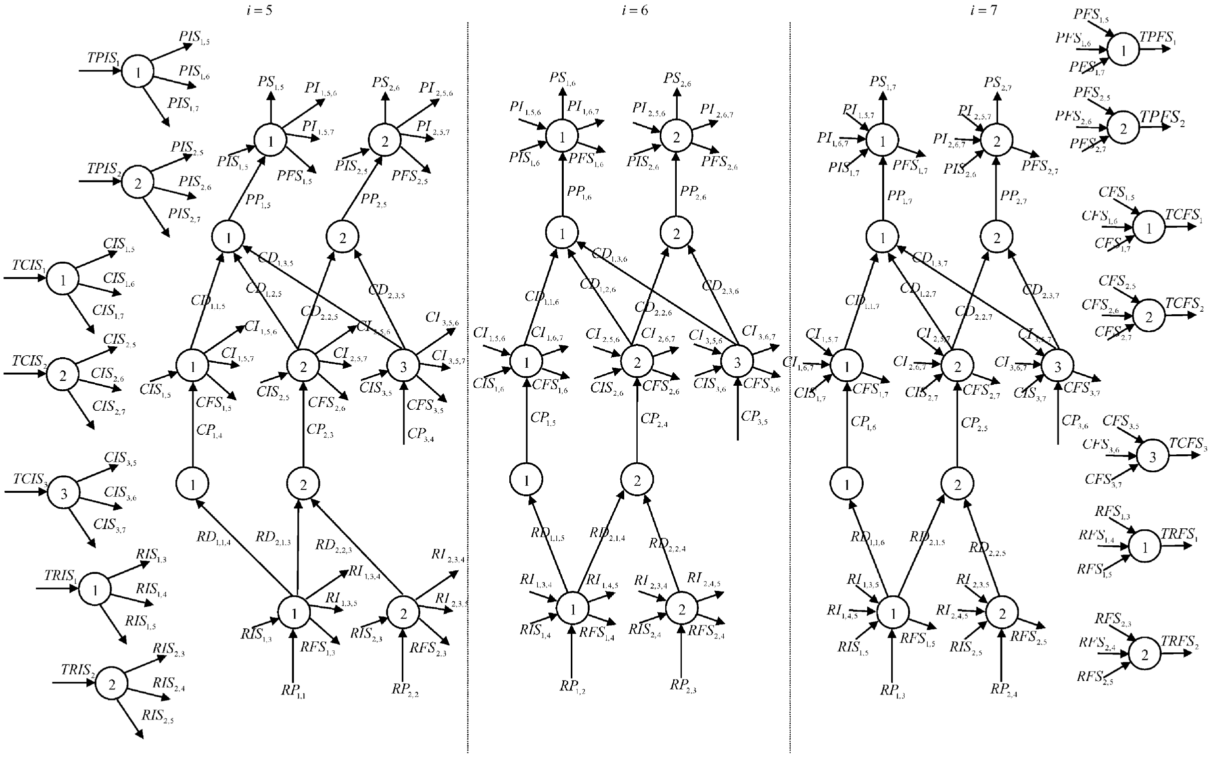

Fig. 3. The flow network of the example case.

Table 1
Production/procurement volumes of the materials

Materials		Periods						
		1	2	3	4	5	6	7
Products	1	N/A	N/A	N/A	N/A	7000	10,000	13,000
	2	N/A	N/A	N/A	N/A	14,500	18,000	21,500
Components	1	N/A	N/A	N/A	N/A	0	8000	22,000
	2	N/A	N/A	33,500	56,000	78,500	N/A	N/A
	3	N/A	N/A	N/A	57,000	82,000	107,000	N/A
Raw materials	1	3000	9600	19,200	N/A	N/A	N/A	N/A
	2	N/A	1,275	2800	0	N/A	N/A	N/A
		Objective function value = 12,209,890						

N/A: The value cannot exist due to the sample problem's structure.

The optimum values of the basic decision variables are summarized in the Table 1. The signs of "N/A" in the cells of the Table 1 mean that the supply for the intersection of corresponding period and item cannot exist due to the structure of the sample problem. The complete solution is provided in Appendix B.

5. Conclusion and future research

In this paper, a model is given in order to show that the flow-network approach can be applied to the MRP problems. The flow-network approach is used since materials can physically flow from one point to another, and in the same way from one moment to another. Nodes represent the planning moments and materials while arcs stand for the material flows. This approach is very suitable to express the material flow logic behind the MRP since materials flow within a production system. The flow network approach easily visualizes these flows. Moreover, the material flows among the planning periods can be visualized.

The pure or generalized flow network without side constraints approach is not suitable to the MRP problems. Because, there are proportional relations exist among consecutive levels in the hierarchy of materials in the BOM. Therefore, the flow network with side constraints, or in other words the embedded generalized flow network, formulation is used for the model.

In the model for this research, all mathematical relations are expressed in the linear form. Jensen and Barnes (1987) give information about a minimal cost flow problem can be extended to include nonlinear feature. In their example, they discuss the electrical current flow in wires. The network has nonlinearities due to the nonlinear voltage characteristics of the various electronic elements like diodes and nonlinear resistors. Additionally, positive and negative currents cause nonlinearities too. They define the content of an element for the given current by the area under the voltage characteristic curve. According to Dembo, Mulvey and Zenios (1989), nonlinearities arise due to physical phenomena and economic considerations. They discuss the sources of nonlinearities in their study. One of the economic and social causes for nonlinearities is that an increase in the volume often decreases marginal costs. Similarly, the cost function of an inventory model may have nonlinear aspects. For example, as Jorjani and Lamar (1994) discuss in their paper, a discount according to the lot size leads to the nonlinearities in the cost function. The price may change according to a nonlinear function of the amount to purchase. Since the sum of the money to be paid for buying the items is directly in the cost function, it will become nonlinear.

The research given in this article can extend to include some stochastic features. Jensen and Barnes (1987) also discuss the cost functions that depend on random variables in their book. They define the expected cost as a function of flow when an arc has a cost function depends both on flow and on some random variable, which has a probability density function. Furthermore, they define a penalty function based on a random variable so that a flow to a certain arc does not meet a demand. This situation can be applied to the case in which the supply does not meet the requirements. Simultaneously, they prove that the stochastic feature is another cause of nonlinearity. Pourbabai et al. (1996) define the probability of finding at least K_{ij} units along arc (i,j) as a constraint in their model. Such an approach can be applied in order to guarantee the supply sufficiency. Ball et al. (1998) define the stochastic model so that "Each arc e operates with the probability p_e and fails with a probability $1-p_e$. When an arc operates, it has unit capacity; when it fails, it can carry no flow." Grubbström and Wang (2003) take stochastic demand feature into consideration. In their model, stochastic demand events are assumed to take place at discrete time points.

The problems in real world are not always strictly deterministic because of the existing uncertainties and risks. Mostly, they have probabilistic characteristics. Therefore, it is an important research field to implement the stochastic property of the problems to the model given and discussed in this article. Moreover, these two features, to be stochastic and nonlinear, can be combined in order to obtain more realistic models.

Another further research area for this study would be the lot sizing rules. In this study, the lot-for-lot (L4L) approach, which is one of the dynamic lot sizing rules, is used. Plossl (1994), Orlicky (1975) and Heisig (2002) say that the L4L rule minimizes the inventory holding cost and is good for the schedule stability. However, one can claim that other lot sizing methods would be better from the cost efficiency point of view. van Donselaar and Gubbels (2002) claim in their conclusion that fixed lot sizes should be used rather than dynamic lot sizes in order to minimize system nervousness. It is obvious that the adoption of predefined lot sizes will lead to the necessity of the mixed-integer programming approach. In such a case, the solution time could be very long since the formulation would become NP complete. Only the heuristic algorithms can solve NP complete problems.

The approach given in this paper can be embedded into an integrated planning system as an advanced planning and scheduling (APS) system. The advanced planning and scheduling system is a kind of system running background and performing some optimization operations. Norris et al. (2000) debate that the APS system provides value in three main areas: (1) Constraint-based planning, (2) Real-time processing and (3) Integration. Stadtler and Kilger (2002) give three main characteristics of APS; integral planning, true optimization and a hierarchical planning system. They argue that traditional MRP concept does not have these properties and does not optimize. The flow-network approach given in this study has an optimization characteristic. MRP is a mid-term or short-term procurement task in the supply chain planning matrix. Therefore, it is a part of the supply chain. Naturally, a good implementation of the formulation mentioned in this study into an APS requires highly integration among the components of the system, since they continuously interact with each other. Obviously, the data from other parts of the system will feed this formulation if it is implemented in an APS. Moreover, this formulation will generate outputs to feed the other parts. The parts of the APS interact with each other via interfaces. A few examples of the actual ASPs and case studies are given in Stadtler and Kilger (2002). However, only seldom does APS providers launch a special module for procurement decisions directly. Therefore, the formulation given in this study will help the researchers studying on the development of such an "advanced" module. Gayialis and Tatsiopoulos (2004) claim that OR algorithms can be applied in practice in the form of advanced planning and scheduling system if they are embedded in packaged information technology solutions and the interface problems to mainstream ERP software applications are solved. Additionally, they introduce such a decision support system for vehicle routing and scheduling, and discuss the benefits gained. Similarly, the formulation developed in this study can be embedded into an APS as an optimization module.

Appendix A. The complete model of the sample problem

$$\begin{aligned}
&P = \{1,2\}\\
&C = \{1,2,3\}\\
&R = \{1,2\}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{BOMP} = \big\{(1,1),(1,2)(1,3),(2,2),(2,3)\big\}\\
&\mathrm{BOMC} = \big\{(1,1)(2,1),(2,2)\big\}\\
&\mathrm{CL}_1 = 1,\ \ \mathrm{CL}_2 = 2,\ \ \mathrm{CL}_3 = 1\\
&\mathrm{RL}_1 = 2,\ \ \mathrm{RL}_2 = 1
\end{aligned}$$

$$\begin{aligned}
\min I &= \max\big\{\mathrm{CL}_c + \mathrm{RL}_r | (c,r) \in \mathrm{BOMC}\big\} + 1\\
&= \max\{\mathrm{CL}_1 + \mathrm{RL}_1, \mathrm{CL}_2 + \mathrm{RL}_1, \mathrm{CL}_2 + \mathrm{RL}_2\} + 1\\
&= \max\{3,4,3\} + 1 = 5
\end{aligned}$$

$I = \{5,6,7\}$ (Since the planning horizon consists of three periods)

$$k_1 = 2,\ \ k_2 = 2$$

$$\begin{aligned}
\min\ & \mathrm{pc}_1\mathrm{PP}_{1,5} + \mathrm{pc}_1\mathrm{PP}_{1,6} + \mathrm{pc}_1\mathrm{PP}_{1,7} + \mathrm{pc}_2\mathrm{PP}_{2,5} + \mathrm{pc}_2\mathrm{PP}_{2,6} + \mathrm{pc}_2\mathrm{PP}_{2,7}\\
&+ \mathrm{cc}_1\mathrm{CP}_{1,4} + \mathrm{cc}_1\mathrm{CP}_{1,5} + \mathrm{cc}_1\mathrm{CP}_{1,6} + \mathrm{cc}_2\mathrm{CP}_{2,3} + \mathrm{cc}_2\mathrm{CP}_{2,4} + \mathrm{cc}_2\mathrm{CP}_{2,5}\\
&\quad + \mathrm{cc}_3\mathrm{CP}_{3,4} + \mathrm{cc}_3\mathrm{CP}_{3,5} + \mathrm{cc}_3\mathrm{CP}_{3,6}\\
&+ \mathrm{rc}_1\mathrm{RP}_{1,1} + \mathrm{rc}_1\mathrm{RP}_{1,2} + \mathrm{rc}_1\mathrm{RP}_{1,3} + \mathrm{rc}_2\mathrm{RP}_{2,2} + \mathrm{rc}_2\mathrm{RP}_{2,3} + \mathrm{rc}_2\mathrm{RP}_{2,4}\\
&+ \mathrm{ph}_1\mathrm{PI}_{1,5,6} + 2\mathrm{ph}_1\mathrm{PI}_{1,5,7} + \mathrm{ph}_1\mathrm{PI}_{1,6,7} + \mathrm{ph}_2\mathrm{PI}_{2,5,6} + 2\mathrm{ph}_2\mathrm{PI}_{2,5,7} + \mathrm{ph}_2\mathrm{PI}_{2,6,7}\\
&+ \mathrm{ch}_1\mathrm{CI}_{1,5,6} + 2\mathrm{ch}_1\mathrm{CI}_{1,5,7} + \mathrm{ch}_1\mathrm{CI}_{1,6,7} + \mathrm{ch}_2\mathrm{CI}_{2,5,6} + 2\mathrm{ch}_2\mathrm{CI}_{2,5,7} + \mathrm{ch}_2\mathrm{CI}_{2,6,7}\\
&\quad + \mathrm{ch}_3\mathrm{CI}_{3,5,6} + 2\mathrm{ch}_3\mathrm{CI}_{3,5,7} + \mathrm{ch}_3\mathrm{CI}_{3,6,7}\\
&+ \mathrm{rh}_1\mathrm{RI}_{1,3,4} + 2\mathrm{rh}_1\mathrm{RI}_{1,3,5} + \mathrm{rh}_1\mathrm{RI}_{1,4,5} + \mathrm{rh}_2\mathrm{RI}_{2,3,4} + 2\mathrm{rh}_2\mathrm{RI}_{2,3,5} + \mathrm{rh}_2\mathrm{RI}_{2,4,5}\\
&+ 3\mathrm{ph}_1\mathrm{PFS}_{1,5} + 2\mathrm{ph}_1\mathrm{PFS}_{1,6} + \mathrm{ph}_1\mathrm{PFS}_{1,7} + 3\mathrm{ph}_2\mathrm{PFS}_{2,5} + 2\mathrm{ph}_2\mathrm{PFS}_{2,6} + \mathrm{ph}_2\mathrm{PFS}_{2,7}\\
&+ 3\mathrm{ch}_1\mathrm{CFS}_{1,5} + 2\mathrm{ch}_1\mathrm{CFS}_{1,6} + \mathrm{ch}_1\mathrm{CFS}_{1,7} + 3\mathrm{ch}_2\mathrm{CFS}_{2,5} + 2\mathrm{ch}_2\mathrm{CFS}_{2,6} + \mathrm{ch}_2\mathrm{CFS}_{2,7}\\
&\quad + 3\mathrm{ch}_3\mathrm{CFS}_{3,5} + 2\mathrm{ch}_3\mathrm{CFS}_{3,6} + \mathrm{ch}_3\mathrm{CFS}_{3,7}\\
&+ 3\mathrm{rh}_1\mathrm{RFS}_{1,3} + 2\mathrm{rh}_1\mathrm{RFS}_{1,4} + \mathrm{rh}_1\mathrm{RFS}_{1,5} + 3\mathrm{rh}_2\mathrm{RFS}_{2,3} + 2\mathrm{rh}_2\mathrm{RFS}_{2,4} + \mathrm{rh}_2\mathrm{RFS}_{2,5}\\
&+ \mathrm{ph}_1\mathrm{PIS}_{1,6} + 2\mathrm{ph}_1\mathrm{PIS}_{1,7} + \mathrm{ph}_2\mathrm{PIS}_{2,6} + 2\mathrm{ph}_2\mathrm{PIS}_{2,7}\\
&+ \mathrm{ch}_1\mathrm{CIS}_{1,6} + 2\mathrm{ch}_1\mathrm{CIS}_{1,7} + \mathrm{ch}_2\mathrm{CIS}_{2,6} + 2\mathrm{ch}_2\mathrm{CIS}_{2,7} + \mathrm{ch}_3\mathrm{CIS}_{3,6} + 2\mathrm{ch}_3\mathrm{CIS}_{3,7}\\
&+ \mathrm{rh}_1\mathrm{RIS}_{1,4} + 2\mathrm{rh}_1\mathrm{RIS}_{1,5} + \mathrm{rh}_2\mathrm{RIS}_{2,4} + 2\mathrm{rh}_2\mathrm{RIS}_{2,5}\\
&+ \mathrm{pih}_1\mathrm{TPIS}_1 + \mathrm{pih}_2\mathrm{TPIS}_2\\
&+ \mathrm{cih}_1\mathrm{TCIS}_1 + \mathrm{cih}_2\mathrm{TCIS}_2 + \mathrm{cih}_3\mathrm{TCIS}_3\\
&+ \mathrm{rih}_1\mathrm{TRIS}_1 + \mathrm{rih}_2\mathrm{TRIS}_2.
\end{aligned}$$

Subject to

$$
\begin{aligned}
&\mathrm{PS}_{1,5}+\mathrm{PI}_{1,5,6}+\mathrm{PI}_{1,5,7}+\mathrm{PFS}_{1,5}-\mathrm{PP}_{1,5}-\mathrm{PIS}_{1,5}=0\\
&\mathrm{PS}_{2,5}+\mathrm{PI}_{2,5,6}+\mathrm{PI}_{2,5,7}+\mathrm{PFS}_{2,5}-\mathrm{PP}_{2,5}-\mathrm{PIS}_{2,5}=0\\
&\mathrm{PS}_{1,6}+\mathrm{PI}_{1,6,7}+\mathrm{PFS}_{1,6}-\mathrm{PI}_{1,5,6}-\mathrm{PP}_{1,6}-\mathrm{PIS}_{1,6}=0\\
&\mathrm{PS}_{2,6}+\mathrm{PI}_{2,6,7}+\mathrm{PFS}_{2,6}-\mathrm{PI}_{2,5,6}-\mathrm{PP}_{2,6}-\mathrm{PIS}_{2,6}=0\\
&\mathrm{PS}_{1,7}+\mathrm{PFS}_{1,7}-\mathrm{PI}_{1,5,7}-\mathrm{PI}_{1,6,7}-\mathrm{PP}_{1,7}-\mathrm{PIS}_{1,7}=0\\
&\mathrm{PS}_{2,7}+\mathrm{PFS}_{2,7}-\mathrm{PI}_{2,5,7}-\mathrm{PI}_{2,6,7}-\mathrm{PP}_{2,7}-\mathrm{PIS}_{2,7}=0
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{PS}_{1,5}\geqslant\mathrm{PD}_{1,5}\\
&\mathrm{PS}_{2,5}\geqslant\mathrm{PD}_{2,5}\\
&\mathrm{PS}_{1,6}\geqslant\mathrm{PD}_{1,6}\\
&\mathrm{PS}_{2,6}\geqslant\mathrm{PD}_{2,6}\\
&\mathrm{PS}_{1,7}\geqslant\mathrm{PD}_{1,7}\\
&\mathrm{PS}_{2,7}\geqslant\mathrm{PD}_{2,7}
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{PIS}_{1,5}+\mathrm{PIS}_{1,6}+\mathrm{PIS}_{1,7}-\mathrm{TPIS}_{1}=0\\
&\mathrm{PIS}_{2,5}+\mathrm{PIS}_{2,6}+\mathrm{PIS}_{2,7}-\mathrm{TPIS}_{2}=0\\
&-\mathrm{PFS}_{1,5}-\mathrm{PFS}_{1,6}-\mathrm{PFS}_{1,7}+\mathrm{TPFS}_{1}=0\\
&-\mathrm{PFS}_{2,5}-\mathrm{PFS}_{2,6}-\mathrm{PFS}_{2,7}+\mathrm{TPFS}_{2}=0
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{CD}_{1,1,5}-\mathrm{cu}_{1,1}\mathrm{PP}_{1,5}=0\\
&\mathrm{CD}_{1,2,5}-\mathrm{cu}_{1,2}\mathrm{PP}_{1,5}=0\\
&\mathrm{CD}_{1,3,5}-\mathrm{cu}_{1,3}\mathrm{PP}_{1,5}=0\\
&\mathrm{CD}_{2,2,5}-\mathrm{cu}_{2,2}\mathrm{PP}_{2,5}=0\\
&\mathrm{CD}_{2,3,5}-\mathrm{cu}_{2,3}\mathrm{PP}_{2,5}=0\\
&\mathrm{CD}_{1,1,6}-\mathrm{cu}_{1,1}\mathrm{PP}_{1,6}=0\\
&\mathrm{CD}_{1,2,6}-\mathrm{cu}_{1,2}\mathrm{PP}_{1,6}=0\\
&\mathrm{CD}_{1,3,6}-\mathrm{cu}_{1,3}\mathrm{PP}_{1,6}=0\\
&\mathrm{CD}_{2,2,6}-\mathrm{cu}_{2,2}\mathrm{PP}_{2,6}=0\\
&\mathrm{CD}_{2,3,6}-\mathrm{cu}_{2,3}\mathrm{PP}_{2,6}=0\\
&\mathrm{CD}_{1,1,7}-\mathrm{cu}_{1,1}\mathrm{PP}_{1,7}=0\\
&\mathrm{CD}_{1,2,7}-\mathrm{cu}_{1,2}\mathrm{PP}_{1,7}=0\\
&\mathrm{CD}_{1,3,7}-\mathrm{cu}_{1,3}\mathrm{PP}_{1,7}=0\\
&\mathrm{CD}_{2,2,7}-\mathrm{cu}_{2,2}\mathrm{PP}_{2,7}=0\\
&\mathrm{CD}_{2,3,7}-\mathrm{cu}_{2,3}\mathrm{PP}_{2,7}=0
\end{aligned}
$$

$$\begin{aligned}
&\mathrm{CD}_{1,1,5}+\mathrm{CI}_{1,5,6}+\mathrm{CI}_{1,5,7}+\mathrm{CFS}_{1,5}-\mathrm{CP}_{1,4}-\mathrm{CIS}_{1,5}=0\\
&\mathrm{CD}_{1,2,5}+\mathrm{CD}_{2,2,5}+\mathrm{CI}_{2,5,6}+\mathrm{CI}_{2,5,7}+\mathrm{CFS}_{2,5}-\mathrm{CP}_{2,3}-\mathrm{CIS}_{2,5}=0\\
&\mathrm{CD}_{1,3,5}+\mathrm{CD}_{2,3,5}+\mathrm{CI}_{3,5,6}+\mathrm{CI}_{3,5,7}+\mathrm{CFS}_{3,5}-\mathrm{CP}_{3,4}-\mathrm{CIS}_{3,5}=0\\
&\mathrm{CD}_{1,1,6}+\mathrm{CI}_{1,6,7}+\mathrm{CFS}_{1,6}-\mathrm{CI}_{1,5,6}-\mathrm{CP}_{1,5}-\mathrm{CIS}_{1,6}=0\\
&\mathrm{CD}_{1,2,6}+\mathrm{CD}_{2,2,6}+\mathrm{CI}_{2,6,7}+\mathrm{CFS}_{2,6}-\mathrm{CI}_{2,5,6}-\mathrm{CP}_{2,4}-\mathrm{CIS}_{2,6}=0\\
&\mathrm{CD}_{1,3,6}+\mathrm{CD}_{2,3,6}+\mathrm{CI}_{3,6,7}+\mathrm{CFS}_{3,6}-\mathrm{CI}_{3,5,6}-\mathrm{CP}_{3,5}-\mathrm{CIS}_{3,6}=0\\
&\mathrm{CD}_{1,1,7}+\mathrm{CFS}_{1,7}-\mathrm{CI}_{1,5,7}-\mathrm{CI}_{1,6,7}-\mathrm{CP}_{1,6}-\mathrm{CIS}_{1,7}=0\\
&\mathrm{CD}_{1,2,7}+\mathrm{CD}_{2,2,7}+\mathrm{CFS}_{2,7}-\mathrm{CI}_{2,5,7}-\mathrm{CI}_{2,6,7}-\mathrm{CP}_{2,5}-\mathrm{CIS}_{2,7}=0\\
&\mathrm{CD}_{1,3,7}+\mathrm{CD}_{2,3,7}+\mathrm{CFS}_{3,7}-\mathrm{CI}_{3,5,7}-\mathrm{CI}_{3,6,7}-\mathrm{CP}_{3,6}-\mathrm{CIS}_{3,7}=0
\end{aligned}$$

$$\begin{aligned}
&\mathrm{CIS}_{1,5}+\mathrm{CIS}_{1,6}+\mathrm{CIS}_{1,7}-\mathrm{TCIS}_{1}=0\\
&\mathrm{CIS}_{2,5}+\mathrm{CIS}_{2,6}+\mathrm{CIS}_{2,7}-\mathrm{TCIS}_{2}=0\\
&\mathrm{CIS}_{3,5}+\mathrm{CIS}_{3,6}+\mathrm{CIS}_{3,7}-\mathrm{TCIS}_{3}=0
\end{aligned}$$

$$\begin{aligned}
&-\mathrm{CFS}_{1,5}-\mathrm{CFS}_{1,6}-\mathrm{CFS}_{1,7}+\mathrm{TCFS}_{1}=0\\
&-\mathrm{CFS}_{2,5}-\mathrm{CFS}_{2,6}-\mathrm{CFS}_{2,7}+\mathrm{TCFS}_{2}=0\\
&-\mathrm{CFS}_{3,5}-\mathrm{CFS}_{3,6}-\mathrm{CFS}_{3,7}+\mathrm{TCFS}_{3}=0
\end{aligned}$$

$$\begin{aligned}
&\mathrm{RD}_{1,1,4}-\mathrm{ru}_{1,1}\mathrm{CP}_{1,4}=0\\
&\mathrm{RD}_{2,1,3}-\mathrm{ru}_{2,1}\mathrm{CP}_{2,3}=0\\
&\mathrm{RD}_{2,2,3}-\mathrm{ru}_{2,2}\mathrm{CP}_{2,3}=0\\
&\mathrm{RD}_{1,1,5}-\mathrm{ru}_{1,1}\mathrm{CP}_{1,5}=0\\
&\mathrm{RD}_{2,1,4}-\mathrm{ru}_{2,1}\mathrm{CP}_{2,4}=0\\
&\mathrm{RD}_{2,2,4}-\mathrm{ru}_{2,2}\mathrm{CP}_{2,4}=0\\
&\mathrm{RD}_{1,1,6}-\mathrm{ru}_{1,1}\mathrm{CP}_{1,6}=0\\
&\mathrm{RD}_{2,1,5}-\mathrm{ru}_{2,1}\mathrm{CP}_{2,5}=0\\
&\mathrm{RD}_{2,2,5}-\mathrm{ru}_{2,2}\mathrm{CP}_{2,5}=0
\end{aligned}$$

$$\begin{aligned}
&\mathrm{RD}_{1,1,4}+\mathrm{RD}_{2,1,3}+\mathrm{RI}_{1,3,4}+\mathrm{RI}_{1,3,5}+\mathrm{RFS}_{1,3}-\mathrm{RP}_{1,1}-\mathrm{RIS}_{1,3}=0\\
&\mathrm{RD}_{2,2,3}+\mathrm{RI}_{2,3,4}+\mathrm{RI}_{2,3,5}+\mathrm{RFS}_{2,3}-\mathrm{RP}_{2,2}-\mathrm{RIS}_{2,3}=0\\
&\mathrm{RD}_{1,1,5}+\mathrm{RD}_{2,1,4}+\mathrm{RI}_{1,4,5}+\mathrm{RFS}_{1,4}-\mathrm{RI}_{1,3,4}-\mathrm{RP}_{1,2}-\mathrm{RIS}_{1,4}=0\\
&\mathrm{RD}_{2,2,4}+\mathrm{RI}_{2,4,5}+\mathrm{RFS}_{2,4}-\mathrm{RI}_{2,3,4}-\mathrm{RP}_{2,3}-\mathrm{RIS}_{2,4}=0\\
&\mathrm{RD}_{1,1,6}+\mathrm{RD}_{2,1,5}+\mathrm{RFS}_{1,5}-\mathrm{RI}_{1,3,5}-\mathrm{RI}_{1,4,5}-\mathrm{RP}_{1,3}-\mathrm{RIS}_{1,5}=0\\
&\mathrm{RD}_{2,2,5}+\mathrm{RFS}_{2,5}-\mathrm{RI}_{2,3,5}-\mathrm{RI}_{2,4,5}-\mathrm{RP}_{2,4}-\mathrm{RIS}_{2,5}=0
\end{aligned}$$

$$\begin{aligned}
&\mathrm{RIS}_{1,3}+\mathrm{RIS}_{1,4}+\mathrm{RIS}_{1,5}-\mathrm{TRIS}_{1}=0\\
&\mathrm{RIS}_{2,3}+\mathrm{RIS}_{2,4}+\mathrm{RIS}_{2,5}-\mathrm{TRIS}_{2}=0
\end{aligned}$$

$$\begin{aligned}
&-\mathrm{RFS}_{1,3}-\mathrm{RFS}_{1,4}-\mathrm{RFS}_{1,5}+\mathrm{TRFS}_{1}=0\\
&-\mathrm{RFS}_{2,3}-\mathrm{RFS}_{2,4}-\mathrm{RFS}_{2,5}+\mathrm{TRFS}_{2}=0.
\end{aligned}$$

Appendix B. The complete solution of the sample problem

Objective function value
1) 12209890

Variable	Value	Variable	Value	Variable	Value
PP15	7000	PFS16	0	PS25	18000
PP16	10000	PFS17	3000	PIS25	3500
PP17	13000	PFS25	0	PS16	10000
PP25	14500	PFS26	0	PS26	18000
PP26	18000	PFS27	3500	PS17	10000
PP27	21500	CFS15	0	PS27	18000
CP15	0	CFS16	0	TPFS1	3000
CP16	8000	CFS17	9000	TPFS2	3500
CP17	22000	CFS25	0	CD115	7000
CP23	33500	CFS26	0	CD125	14000
CP24	56000	CFS27	9500	CD135	7000
CP25	78500	CFS35	0	CD225	29000
CP34	57000	CFS36	0	CD235	58000
CP35	82000	CFS37	8000	CD116	10000
CP36	107000	RFS13	0	CD126	20000
RP11	3000	RFS14	0	CD136	10000
RP12	9600	RFS15	350	CD226	36000
RP13	19200	RFS23	0	CD236	72000
RP22	1275	RFS24	0	CD117	13000
RP23	2800	RFS25	400	CD127	26000
RP24	0	PIS16	0	CD137	13000
PI156	0	PIS17	0	CD227	43000
PI157	0	PIS26	0	CD237	86000
PI167	0	PIS27	0	CIS15	9000
PI256	0	CIS16	0	CIS25	9500
PI257	0	CIS17	0	CIS35	8000
PI267	0	CIS26	0	TCFS1	9000
CI156	2000	CIS27	0	TCFS2	9500
CI157	0	CIS36	0	TCFS3	8000
CI167	0	CIS37	0	RD114	0
CI256	0	RIS14	0	RD213	3350
CI257	0	RIS15	0	RD223	1675
CI267	0	RIS24	0	RD115	4000
CI356	0	RIS25	0	RD214	5600
CI357	0	TPIS1	3000	RD224	2800
CI367	0	TPIS2	3500	RD116	11000
RI134	0	TCIS1	9000	RD215	7850
RI135	0	TCIS2	9500	RD225	3925
RI145	0	TCIS3	8000	RIS13	350
RI234	0	TRIS1	350	RIS23	400
RI235	0	TRIS2	400	RI244	0

RI245	4325	PS15	10000	TRFS1	350
PFS15	0	PIS15	3000	TRFS2	400

No of Iterations = 73

References

Ali, A.I., Kennington, J., Shetty, B., 1988. The equal flow problem. European Journal of Operational Research 36, 107–115.

Ball, M.O., Hagstrom, J.N., Provan, J.S., 1998. Two-path subsets: Efficient counting and applications to performability analysis. Discrete Applied Mathematics 85, 25–45.

Chu, S.C.K., 1995. A mathematical programming approach towards optimized master production scheduling. International Journal of Production Economics 38, 269–279.

Clark, A.R., 2003. Optimization approximations for capacity constrained material requirements planning. International Production Economics 84, 115–131.

Dembo, R.S., Mulvey, J.M., Zenios, S.A., 1989. Large-scale nonlinear network models and their applications. Operations Research 37 (3), 353–372.

Euwe, M.J., Wortmann, H., 1997. Planning systems in the next century (I), Computers in Industry 34, 233–237.

Gayialis, S.P., Tatsiopoulos, I.P., 2004. Design of an IT-driven decision support system for vehicle routing and scheduling. European Journal of Operational Research 152, 382–398.

Glover, F., Klingman, D., Phillips, N.V., 1992. Network models in Optimization and Their Applications in Practice. Wiley, New York.

Grubbström, R.W., Wang, Z., 2003. A stochastic model of multi-level/multi-stage capacity-constrained production-inventory systems. International Journal of Production Economics 81–82, 483–494.

Heisig, G., 2002. Planning Stability in Material Requirements Planning Systems. Springer, Berlin Heidelberg.

Jensen, P. A., Barnes, J. W., 1987. In: Robert E. (Ed.), Network Flow Programming. Krieger Publishing Company, Malabar Florida.

Jorjani, S., Lamar, B.W., 1994. Cash flow management network models with quantity discounting, omega. International Journal of Management Science 22 (2), 149–155.

Mathies, S., Mevert, P., 1998. A Hybrid algorithm for solving network flow problems with side constraints. Computers and Operations Research 25 (9), 745–756.

McBride, R.D., 1985. Solving embedded generalized network problems. European Journal of Operational Research 21, 82–92.

Norris, G., Hurley, J.R., Hartley, K.M., Dunleavy, J.R., Balls, J.D., 2000. E-business and ERP: Transforming the Enterprise. Wiley, New York.

Orlicky, J., 1975. Material Requirements Planning. McGraw-Hill Book Company, New York.

Plenert, G., 1999. Focusing material requirements planning (MRP) towards performance. European Journal of Operational Research 119, 91–99.

Plossl, G., 1994. Orlicky's Material Requirements Planning, second ed. McGraw-Hill Inc., New York.

Pourbabai, B., Blanc, J.P.C., van der Duyn Schouten, 1996. European Journal of Operational Research 88, 586–591.

Rondeau, P.J., Litteral, L.A., 2001. Evolution of manufacturing planning and control systems: From reorder point to enterprise resource planning. Production and Inventory Management Journal second quarter, 1–7.

Segerstedt, A., 1996. Formulas of MRP. International Journal of Production Economics 46–47, 127–136.

Stadtler, H., Kilger, C., 2002. Supply Chain Management and Advanced Planning; Concepts, Models, Software and Case Studies, second ed. Springer, Berlin Heidelberg.

van Donselaar, K.H., Gubbels, B.J., 2002. How to release orders in order to minimise system inventory and system nervousness? International Journal of Production Economics 78, 335–343.

Yenisey, M. M., 1998. Flow-network approach to materials requirement planning. Proceedings of the second International Symposium on Intelligent Manufacturing Systems, Vol.1, August 6 and 7, 1998, Sakarya-TURKEY, pp. 529–533.

Zijm, W.H.M., 2000. Towards Intelligent Manufacturing Planning and Control Systems. OR Spektrum 22, 313–345.